University of California, Berkeley
Physics 110A, Section 2, Spring 2003 (*Strovink*)

PROBLEM SET 5

1.
Griffiths Problem 4.28.

2.
In vacuum a large number of nonrelativistic particles, all with the same |charge| e and mass m, are created at the origin. Further, all particles have the same nonzero initial z component of velocity, v_{z0}. Their other initial velocity components vary randomly. The vacuum is filled with a uniform magnetic field $\vec{B} = B_0\hat{z}$. Show that all particles pass through at least one common coordinate (x, y, z) other than the origin, and find that coordinate.

[This *focussing property* of solenoidal magnetic fields is widely used to create intense beams of charged particles, particularly muons.]

3.
Griffiths' Ex. 5.2 obtains a cycloid solution for motion in crossed electric and magnetic fields using the Lorentz force law in the laboratory frame. Please read it. Then solve the same problem using a more elegant approach:
(a.)
Nonrelativistically, if frame $\mathcal{S}'$ moves with velocity $\vec{\beta}c$ with respect to (laboratory) frame $\mathcal{S}$, the Galilei transformation for electric fields is

$$\vec{E}\,' = \vec{E} + \vec{\beta} \times c\vec{B} \ .$$

Taking $\vec{B}$ along $\hat{x}$ and $\vec{E}$ along $\hat{z}$, as in the problem, and assuming $E \ll cB$ in magnitude, choose the simplest $\vec{\beta}$ so that $\vec{E}\,'$ vanishes.
(b.)
The two frames coincide at $t = 0$. Using the Galilei transformation for velocities, if the particle is at rest at $t = 0$ in frame $\mathcal{S}$, what is its initial velocity in frame $\mathcal{S}'$?
(c.)
The Galilei transformation for magnetic fields is

$$c\vec{B}\,' = c\vec{B} - \vec{\beta} \times \vec{E} \ .$$

Because $E \ll cB$ and $\beta \ll 1$, approximate $\vec{B}\,'$ to be essentially the same as $\vec{B}$. In frame $\mathcal{S}'$, solve for the motion of the particle.
(d.)
Using the Galilei transformation for positions, determine the motion in the lab frame $\mathcal{S}$.

4.
Consider a beam of charged particles with momentum $\vec{p} = p_0\hat{z}$ and fundamental charge e, incident normally on a "bending magnet". In a region $0 < z < D$, the field of this magnet may be approximated to be uniform, $\vec{B} = B_0\hat{y}$; outside that region, in the path of the beam, it may be approximated to vanish.

Show that a bending magnet imparts a fixed transverse momentum $p_x = p_T$ to every particle, regardless of its initial momentum p_0, so long as the particle is able to pass through the magnet. For $B_0 = 1$ Tesla and $D = 1$ meter, what is p_T in GeV/c?

5.
Consider Ampère's law in vacuum, as modified by Maxwell:

$$\nabla \times \frac{\vec{B}}{\mu_0} = \vec{J} + \frac{\partial}{\partial t}\epsilon_0\vec{E} \ .$$

Taking the divergence of this equation, and applying Gauss's law, prove that electric charge is conserved (Griffiths Eq. 5.29). [Therefore, electric charge conservation is tied fundamentally to the structure of Maxwell's equations; observing the tiniest electric charge nonconservation would completely invalidate the theory.]

6.
A *Helmholtz coil* consists of two circular coils of radius b, each carrying current I in the same sense. The coils are centered on the z axis, parallel to the $z = 0$ plane, and are located at $z = \pm a/2$.

(a.)
Show that all the odd derivatives

$$\frac{d^n B_z}{dz^n} \ , \ n = 1, 3, 5 \ldots$$

vanish at the origin.
(b.)
Find a (in terms of b) such that

$$\frac{d^2 B_z}{dz^2} = 0$$

at the origin.
(c.)
Helmholtz coils often are used in the lab to cancel out the Earth's magnetic field, or to produce a small region of uniform magnetic field in which experiments may be carried out. Along the z axis, suppose that your experiment requires $B_z = 0.01$ T with an error of $< 0.1\%$ over a distance $\Delta z = 0.01$ m. Using an $n \leq 4$ Taylor series expansion, estimate the minimum coil radius b that is needed. Given b, calculate the current I that is required.

7.
Consider two loops of wire, each carrying current I. The first is a circle of radius b in the $z = 0$ plane, centered on the z axis. The second is a square of side $2b$, also in the $z = 0$ plane and centered on the z axis.

The *magnetic dipole moment* $\vec{m}$ of a plane wire loop carrying current I is equal to $I\vec{a}$, where $|a|$ is the loop's area and $\hat{a}$ is the normal to its plane (the ambiguity in $\hat{a}$ is resolved by applying the right-hand rule to the current direction). Far from magnetic dipoles, their magnetic fields drop like r^{-3} (as do electric fields far from electric dipoles).
(a.)
For either of these wire loops, use Ampère's law to show that

$$\int_{-\infty}^{\infty} B_z \, dz = \mu_0 I \ ,$$

where the integral is taken along the z axis.
(b.)
When $z \gg b$, which loop produces the larger $|B_z|$?
(c.)
When z is not $\gg b$, which loop produces the larger $|B_z|$? (You may perform a calculation or you may provide a cogent argument.)